arXiv:1003.3976v1 [cond-mat.mtrl-sci] 21 Mar 2010

Current-induced domain wall motion with adiabatic spin torque only in cylindrical nanowires

Z. Z. Sun* and J. Schliemann
Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany

P. Yan† and X. R. Wang
Physics Department, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong SAR, China

(Dated: March 23, 2010)

We investigate current-driven domain wall (DW) propagation in magnetic nanowires in the framework of the modified Landau-Lifshitz-Gilbert equation with both adiabatic and nonadiabatic spin torque (NAST) terms. Contrary to the common opinion that NAST is indispensable for DW motion[1, 2], we point out that adiabatic spin torque (AST) only is enough for current-driven DW motion in a cylindrical (uniaxial) nanowire. Apart from a discussion of the rigid DW motion from the energy and angular momentum viewpoint, we also propose an experimental scheme to measure the spin current polarization by combining both field and current driven DW motion in a flat (biaxial) wire.

PACS numbers: 75.60.Jk, 75.75.-c, 85.75.-d

The domain wall (DW) motion in magnetic nanowires has recently attracted much attention in the field of nanomagnetism[1] due to the enormous potential industrial applications[3], such as memory bits and logic devices. Besides the field-induced DW motion[4–7], the current-driven magnetization reversal in both magnetic multilayers and nanowires through the spin torque (ST) transfer mechanism[8] has been massively studied, chiefly under the aspect of low power consumption and locality of electric currents. Moreover, a large number of theoretical[2] and experimental studies[9, 10] were devoted to the current-driven DW motion. Here one distinguishes between the adiabatic spin torque (AST) which origins from the polarization of the itinerant electrons adiabatically following the magnetization direction, and the nonadiabatic spin torque (NAST, often referred as a β-term) due to the mismatch of the current polarization and magnetic moments. The common current opinion in the literature is that the latter mechanism is indispensable for current driven DW motion[1, 2].

Experimental studies so far have actually been done on flat nanowires exhibiting a biaxial magnetic anisotropy rather than cylindrical wires realizing the uniaxial case. This is mainly due to difficulties to produce cylindrical metallic wires by conventional lithography although there are techniques available to grow them via electrodeposition[11]. In this letter, starting from an analytical analysis of the modified Landau-Lifshitz-Gilbert (LLG) equation, we show that AST only is enough to drive a sustained DW motion in a cylindrical wire. Our result relies on the fact that the magnetic moments in a cylindrical wire precess around the easy axis, while they rotate in a plane in the biaxial case[5]. Furthermore, in the uniaxial case no Walker breakdown occurs for sufficiently large currents (or fields), as also shown recently by micromagnetic simulations[12]. Finally, we also propose a scheme to measure the spin current polarization P by combining both field and current driven DW motion.

A magnetic nanowire can be described as an effectively one-dimensional (1D) continuum of magnetic moments along the wire axis direction. Magnetic domains are formed due to the competition between the anisotropic magnetic energy and the exchange interaction among adjacent magnetic moments. Without loss of generality, we shall consider a head-to-head DW structure, assuming the easy axis being along the wire (z-) axis. A biaxial anisotropy energy density can be formulated as $\varepsilon^K = -KM_z^2 + K'M_x^2$ where K and K' describe the easy and hard axis anisotropy pointing along the z-axis and the x-axis, respectively. The case $K' = 0$ describes a uniaxial anisotropy along the wire axis. The above energy density is given in units of $\mu_0 M_s^2$, where μ_0 is the vacuum permeability, and $M_s = |\vec{M}|$ such that the local magnetization $\vec{M}$ enters here as a unit vector, $\vec{m} = \vec{M}/M_s$.

If a current is driven through the nanowire, the spatio-temporal magnetization dynamics is governed by the so-called modified LLG equation with additional AST and NAST terms[2],

$$\dot{\vec{m}} = -|\gamma|\vec{m}\times\vec{H}_t + \alpha\vec{m}\times\frac{\partial\vec{m}}{\partial t} - (\vec{u}\cdot\nabla)\vec{m} + \beta\vec{m}\times[(\vec{u}\cdot\nabla)\vec{m}], \quad (1)$$

where $|\gamma|$, α, and β are the gyromagnetic ratio, the Gilbert damping coefficient, and a dimensionless coefficient describing the NAST strength, respectively. β is usually of the same order as the damping in ferromagnetic metals. The velocity $\vec{u}$ points along the flow direction of the itinerant electrons which is usually the wire axis although perpendicular current injection has also been proposed[13]. Thus, $\nabla = \partial/\partial z$ in 1D, and $u = g\mu_B JP/(2eM_s)$ where (among standard notation) J and P are the density and the spin polarization of the current, respectively.

The (normalized) effective field $\vec{h}_t \equiv \vec{H}_t/M_s$ is given by the variational derivative of the total energy density (per unit section-area) $E = \int_{-\infty}^{\infty} dz\varepsilon(z)$ with respect to magnetization, $\vec{h}_t = -\delta E/\delta\vec{m}(z)$. The local energy density is given by[7]

$$\varepsilon(z) = -Km_z^2 + K'm_x^2 + A(\theta'^2 + \sin^2\theta\phi'^2) - \vec{m}\cdot\vec{h}, \quad (2)$$

where A describes the exchange interaction, and $\vec{h}$ is the normalized external field. Moreover, we have adopted the usual spherical coordinates, $\vec{m}(z,t) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ where the polar angle $\theta(z,t)$ and the azimuthal angle $\phi(z,t)$ depend on position and time, and the prime denotes spatial differentiation.

Following Ref. [5, 7], we will focus on DW structures fulfilling $\phi' = \phi'' = 0$, i.e. all the magnetic moments synchronously rotate around the easy axis in space. Then the dynamical LLG equations take the form

$$\begin{aligned}
&\Gamma\dot{\theta} = \alpha(2A\theta'' - K\sin 2\theta - K'\sin 2\theta\cos^2\phi) \\
&\quad + K'\sin\theta\sin 2\phi + (\alpha h_\theta + h_\phi) - (1+\alpha\beta)u\theta', \\
&\Gamma\sin\theta\dot{\phi} = -(2A\theta'' - K\sin 2\theta - K'\sin 2\theta\cos^2\phi) \\
&\quad + \alpha K'\sin\theta\sin 2\phi + (\alpha h_\phi - h_\theta) - (\alpha-\beta)u\theta', \qquad (3)
\end{aligned}$$

where we have defined $\Gamma \equiv 1 + \alpha^2$ and introduced a dimensionless time via $t \mapsto t|\gamma|M_s$. $h_i (i = r, \theta, \phi)$ are the components of the external field in spherical coordinates.

The linear DW motion under field or ST in a biaxial wire has already been discussed in the literature[2, 5]. Let us first review the case of a flat wire with biaxial anisotropy. Following the pioneering work by Schryer and Walker[5] we concentrate on solutions fulfilling $\phi(z,t) \equiv \phi_0 =$ constant. This assumption implies all the magnetic moments move in a plane and is valid at sufficiently low fields[5]. Substituting the travelling-wave ansatz $\tan\frac{\theta}{2} = \exp\left(\frac{z-vt}{\Delta}\right)$ into Eqs. (3), we obtain

$$\begin{aligned}
&\Gamma v = \Delta(\alpha h - K'\sin 2\phi_0) + (1+\alpha\beta)u\,, \\
&\Delta(\alpha K'\sin 2\phi_0 + h) - (\alpha-\beta)u = 0\,,
\end{aligned}$$

where $\Delta \equiv \sqrt{A/(K + K'\cos^2\phi_0)}$ is the DW width, and the external field with a magnitude h along the z axis is assumed ($h_\theta = -h\sin\theta, h_\phi = 0$). Thus, the constant plane angle ϕ_0 and the DW velocity satisfy

$$\alpha K'\sin 2\phi_0 = (\alpha-\beta)u/\Delta - h, \quad v = \Delta h/\alpha + \beta u/\alpha. \quad (4)$$

This solution just recovers the Schryer-Walker result[5] in the presence of ST and implies $\beta \neq 0$ i.e. NAST is indispensable for nonzero DW velocity[2]. In particular, for $|(\alpha-\beta)u/\Delta - h| > \alpha K'$ the sine in Eq. (4) becomes larger than unity, and the solution breaks down. Therefore, the limit of uniaxial anisotropy $K' \to 0$ cannot be reached within the above travelling-wave ansatz.

However, the results (4), devise the following scheme to experimentally determine the spin polarization of the current combining measurements of field-driven and current-driven DW dynamics: First perform velocity measurement using field- and current-driven DWs separately and obtain the quantities $\Delta_{min}/\alpha \equiv C_1$ and $\beta P/\alpha \equiv C_2$. Here $\Delta_{min} = \sqrt{A/(K+K')}$ is the minimum DW width and C_1 can be obtained by extrapolating the data to $h \to 0$. Then apply a fixed field such that the Walker limit is reached and the DW width reaches its maximum, $\Delta_{max} = \sqrt{A/(K+K'/2)}$. By injecting a spin-polarized current and subsequently lowering the current density and monitoring the decrease of DW width one can reach the situation $(\alpha-\beta)u = \Delta h$ implying $\sin 2\phi_0 = 0$ and again $\Delta = \Delta_{min}$. (If $\beta > \alpha$ one may reverse the injected current direction.) Now using $(\alpha-\beta)P/\Delta_{min} \equiv C_3$ one can infer the current polarization $P = C_1C_3 + C_2$ and $\beta/\alpha = C_2/(C_1C_3 + C_2)$. A large anisotropy i.e. $K' \gg K$ and the resolution for observing the DW width variation are the key points for this scheme.

Now we look at the DW motion in a uniaxial wire ($K' = 0$) which is essentially different from the biaxial case. Using the travelling-wave ansatz[7] $\tan\frac{\theta}{2} = \exp\left(\frac{z-vt}{\Delta}\right)$ where now $\Delta = \sqrt{A/K}$, and also for a static field h applied along the z axis, one can straightforwardly obtain the following expressions for the velocity and the precession frequency from Eq.(3),

$$\Gamma v = \alpha\Delta h + (1+\alpha\beta)u, \qquad \Gamma\dot{\phi} = h - (\alpha-\beta)u/\Delta. \quad (5)$$

From Eq. (5), we immediately conclude an important result that sustained DW motion is possible in a cylindrical nanowire even in the absence of NAST, a finding contradicting leading opinions in the recent literature. Moreover, Eq. (5) also shows that, at small damping, the velocity of a current-driven DW is essentially proportional to u while in the field-driven case we have $v \approx \alpha\Delta h$ and the DW motion is suppressed. Thus, in a uniaxial nanowire, a current is more efficient in driving a DW than an external field. Furthermore, the precessional frequency also occurs in the recently studied DW driven electromotive force $V_{emf} = \pm\hbar\dot{\phi}/e$ [14], which should also motivate future experiments. In particular, in a cylindrical nanowire the current polarization can be easily determined experimentally by measuring the velocity of a current-driven DW.

Let us now discuss the energy variation in the field and/or current driven case in both types of nanowires. In general, the change rate of total magnetic energy (from anisotropy, exchange interaction and external magnetic field) can be expressed as

$$\begin{aligned}
\dot{E} &= -\int_{-\infty}^{\infty} dz\dot{\vec{m}}\cdot[\alpha\dot{\vec{m}} + u(\vec{m}\times\partial_z\vec{m}) + \beta u\partial_z\vec{m}] \\
&\equiv P_\alpha + P_{\mathrm{AST}} + P_{\mathrm{NAST}}\,. \qquad (6)
\end{aligned}$$

Here $P_\alpha = -2\alpha\Delta(\dot{\phi}^2 + v^2/\Delta^2)$ is the total dissipation power due to damping, and $P_{\mathrm{AST}} = -2u\dot{\phi}$ and

$P_{\rm NAST} = 2\beta uv/\Delta$ are the energy pumping rates induced by AST and NAST, respectively. Note that the DW width Δ follows different expressions in the biaxial and uniaxial case, and v and $\dot{\phi}$ are given by Eqs. (4) and (5). Remarkably, the AST does not contribute to any change in energy for a biaxial wire ($\dot{\phi} = 0$) below the Walker limit. Moreover, it is straightforward to verify that in both cases it holds $\dot{E} = -2hv$ which is just the released Zeeman energy per time of a rigid DW traveling with velocity v in the external field h[6]. Thus, the relation $\dot{E} = -2hv$ can also be used as a definition of the DW velocity from an energetic point of view. In particular, a moving DW driven purely by current (zero magnetic field) conserves its energy.

Let us now consider the change of angular momentum $\vec{L} = \int dz \vec{m}(z,t)$ (per section-area and in units of $-\mu_0 M_s/|\gamma|$) due to ST. In one hand, the angular momentum change due to rigid DW motion is $\dot{\vec{L}} = \pi\dot{\phi}\Delta\hat{e}_\phi + 2v\hat{z}$. On the other hand, LLG equation (1) gives

$$\begin{aligned} \dot{\vec{L}} &= \int_{-\infty}^{\infty} dz[\vec{h}_t \times \dot{\vec{m}} + \alpha\vec{m} \times \dot{\vec{m}} - u\partial_z\vec{m} - \beta u\vec{m} \times \partial_z\vec{m}] \\ &\equiv \vec{\Gamma}_{pre} + \vec{\Gamma}_\alpha + \vec{\Gamma}_{\rm AST} + \vec{\Gamma}_{\rm NAST}\,. \end{aligned} \tag{7}$$

For our uniaxial wire, $\vec{\Gamma}_{pre} = \pi h\Delta\hat{e}_\phi$, $\vec{\Gamma}_\alpha = -\pi\alpha v\hat{e}_\phi + 2\alpha\dot{\phi}\Delta\hat{z}$, $\vec{\Gamma}_{\rm AST} = 2u\hat{z}$ and $\vec{\Gamma}_{\rm NAST} = \pi\beta u\hat{e}_\phi$. Hence, the AST pumps a longitudinal spin to the wire to push the DW propagating while the NAST and external field provide the transverse torques forcing DW precession around the wire axis. Furthermore, $\vec{\Gamma}_\alpha$ due to the DW precession ($\dot{\phi}$) provides an extra longitudinal torque and that due to the propagation (v) results in an effective transverse force. These two damping effects are reminiscent of Barnett effect[15] and Einstein-de Haas effect[16], respectively. In the present case, both effects originate from non-zero damping $\alpha \neq 0$. Moreover, two of us have recently studied DW motion driven by a circularly polarized microwave, which also embodies the Barnett effect[17]. We also note that Eqs. (5) can equivalently expressed as

$$v = u + \alpha\dot{\phi}\Delta, \qquad \dot{\phi} = h + (\beta u - \alpha v)/\Delta, \tag{8}$$

which shows explicitly that the DW precession contributes to its velocity as $\alpha\dot{\phi}\Delta$, and the DW translation contributes to the precession as $-\alpha v/\Delta$. The above considerations refer to a uniaxial wire. In the biaxial case, however, $\vec{\Gamma}_{pre}$ acquires an additional term from the transverse anisotropy K' and the DW precession is absent. As a result, the effect of AST is canceled and only NAST affects the DW velocity.

Before ending the letter, we would like to give a practical example of the DW motion in a permalloy uniaxial or biaxial nanowire. We employ the standard data M_s = 800kA/m, Δ = 20nm, $\alpha = 0.02, \beta = 0.04, P = 0.4$[2, 4]. For a uniaxial wire, v(m/s)$\approx$ 0.7H(100Oe)+2.9J(10^7A/cm^2), $\dot{\phi}$(GHz) $\approx$ 1.76H(100Oe)+0.003J(10^7A/cm^2); for a flat wire, v(m/s)$\approx$ 1760H(100Oe)+5.8J(10^7A/cm^2) below the Walker limit. Thus the current-induced DW velocity has the same order in both types of wires. The field-driven azimuthal precessional frequency in a cylindrical wire is in GHz regime while current-driven precessional frequency is in MHz regime.

In summary, we have demonstrated, in the framework of the modified Landau-Lifshitz-Gilbert equation, that adiabatic spin torque only is enough for current-driven DW motion in a cylindrical (uniaxial) nanowire. This finding contradicts leading opinions expressed in the recent literature. Moreover, we have also discussed the motion of a rigid DW being subject to (adiabatic or nonadiabatic) spin torque in flat or cylindrical wires from an energetic and angular momentum point of view. Finally, we have also proposed an experimental scheme to measure the spin current polarization by combining both field and current driven DW motion in a flat (biaxial) wire.

Z.Z.S. thanks the Alexander von Humboldt Foundation (Germany) for a grant. This work has been supported by Deutsche Forschugsgemeinschaft via SFB 689. PY and XRW are supported by Hong Kong RGC grants (604109 and HKU10/CRF/08- HKUST17/CRF/08).

* Electronic mail: zhouzhou.sun@physik.uni-regensburg.de
† Electronic mail: yanpeng@ust.hk

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